Inverse problems and uncertainty quantification

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Forward models

Deterministic forward modeling with transient PDE

$$u(t, x; p)$$

known parameters p: initial/boundary data, coefficients, material models, ...

Forward model with uncertainty (posterior)

$$u(t, x, \mathfrak{z}; p)$$

noise $\mathfrak{z}$ is stochastic representation of incompletely-modeled processes

Parameter optimization with uncertainty

$$\min_{p} \int_{\mathfrak{z}} f\big(u(t, x, \mathfrak{z}, p)\big)$$

Inversion and design

Data Assimilation

$$\hat{p}(t, x, d, d_0) = \arg\min_p \int_{\mathfrak{y}} \int_{\mathfrak{z}} \| d(u(t, x, \mathfrak{z}, p) + \mathfrak{y}) - d_0 \|^2 + \mathrm{Prior}(p)$$

infer p from sparse observations d_0 with noise $\mathfrak{y}$

Optimal experimental design

$$\hat{d} = \arg\min_d \int_p \| \hat{p}(t, x, d, d_0(p)) - p \| + \mathrm{cost}(d)$$

choose "affordable" sparse observations d to minimize risk in the data assimilation problem over a region of parameter/model space

Methods for inversion and UQ

- Stochastic Galerkin/collocation
  - Sparse grids, Smolyak quadrature
  - Accurate in low-dimensional spaces
- Bayesian approach

$$\pi_{\text{post}}(x|y_{\text{obs}}) \sim \pi_{\text{prior}}(x)\pi_{\text{like}}(y_{\text{obs}}|x)$$

  - Gaussian prior and linear model: Gaussian posterior
  - Maximum a posteriori (MAP) point (scalable)

$$\min_{x} \ -\log \pi_{\text{post}}(x|y_{\text{obs}})$$

    like regularized deterministic inversion
  - MAP point efficient to compute using adjoints
  - Markov-chain Monte Carlo: high dimensions, nonlinear, but converges slowly