Solving Free Surface Flows for Steady State Without Time Stepping C21A-0503

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Problems with time stepping

The evolution of glaciers and ice sheets occurs on multiple time scales and frequently the slower of these scales are of great scientific importance. Most models of ice sheets and other climate systems are based on methods in which crucial components of the physics are treated explicitly. In addition to reducing the accuracy, time splitting errors produced by such methods may radically change steady states or mispredict hysteresis. With no measure of coupled residual, it is difficult to determine when a system has reached steady state rather than just a period of slow evolution. Furthermore, explicit methods must satisfy stability constraints such that the maximum stable time step is mesh- and parameter-dependent, preventing weak scalability. If the resolution is increased, it is not sufficient to simply run on a larger number of processors since more time steps will be required. For non-stiff hyperbolic equations, it is often desirable to maintain time-accuracy of transport phenomena in which case the CFL condition cannot be circumvented and explicit methods are highly appropriate. Stiff hyperbolic, parabolic, and elliptically constrained equations contain time scales that are not of physical interest and the necessity of explicit methods to resolve these scales prevents scalability.

Implicit methods offer the ability to take time steps independent of mesh resolution, only tracking the time scales of interest. Additionally, bifurcation analysis is most effective when the Jacobian evaluated at a steady state is available, allowing, for example, efficient exploration of a branch jump in multi-dimensional parameter space.

Model equations

Consider the hydrostatic equations (aka. Blatter, Pattyn, "first order") coupled to surface motion and erosion

$$-\nabla\cdot\left[\eta\begin{pmatrix}4u_x+2v_y & u_y+v_x & u_z\\ u_y+v_x & 2u_x+4v_y & v_z\end{pmatrix}\right]+\rho g\nabla s=0$$

$$h_t+\nabla\cdot\int_b^s(u,v)=0$$

$$b_t+k_e\left|(u,v)\right|^p=0$$

where $\eta(\gamma)=\frac{B}{2}(\frac{1}{2}\epsilon^2+\gamma)^{\frac{1-n}{2n}}$ is nonlinear effective viscosity with regularization ϵ and

$$\gamma=u_x^2+v_y^2+u_xv_y+\frac{1}{4}(u_y+v_x)^2+\frac{1}{4}u_z^2+\frac{1}{4}v_z^2$$

is the second invariant. A natural boundary condition is applied at the free surface and slip boundary conditions at the bed with friction coefficient

$$\beta_0^2(\frac{1}{2}\epsilon_b^2+\frac{1}{2}|(u,v)|^2)^{\frac{m-1}{2}}$$

where $m=1$ is linear (Navier) slip, $m=1/3$ is "Weertman sliding", $m\to 0$ is the Coulomb limit, and ϵ_b is regularization which is necessary for $m<1$.

Solution methods

We use a grid-sequenced Newton-Krylov multigrid to solve the coupled equations. An initial guess for the Newton iteration is interpolated from a coarser level, the Newton step is solved by a Krylov iteration preconditioned by geometric multigrid. Our multigrid smoothers use domain decomposition methods with incomplete factorization in an ordering that allows aggressive coarsening.

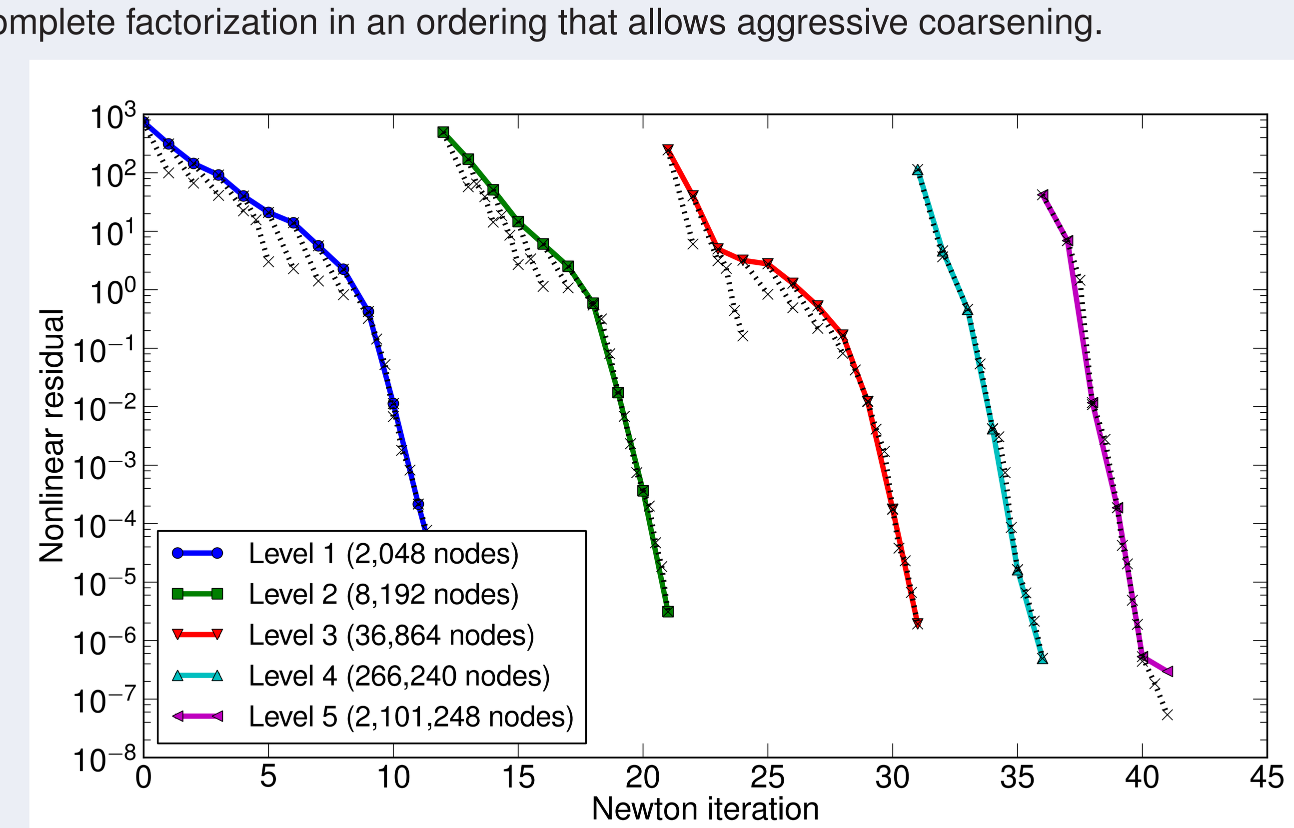

Figure: Grid-sequenced nonlinear convergence on a challenging problem. Colored marks indicate nonlinear residuals, linear residuals are marked with grey ×.

Coupled velocity, surface evolution, and erosion

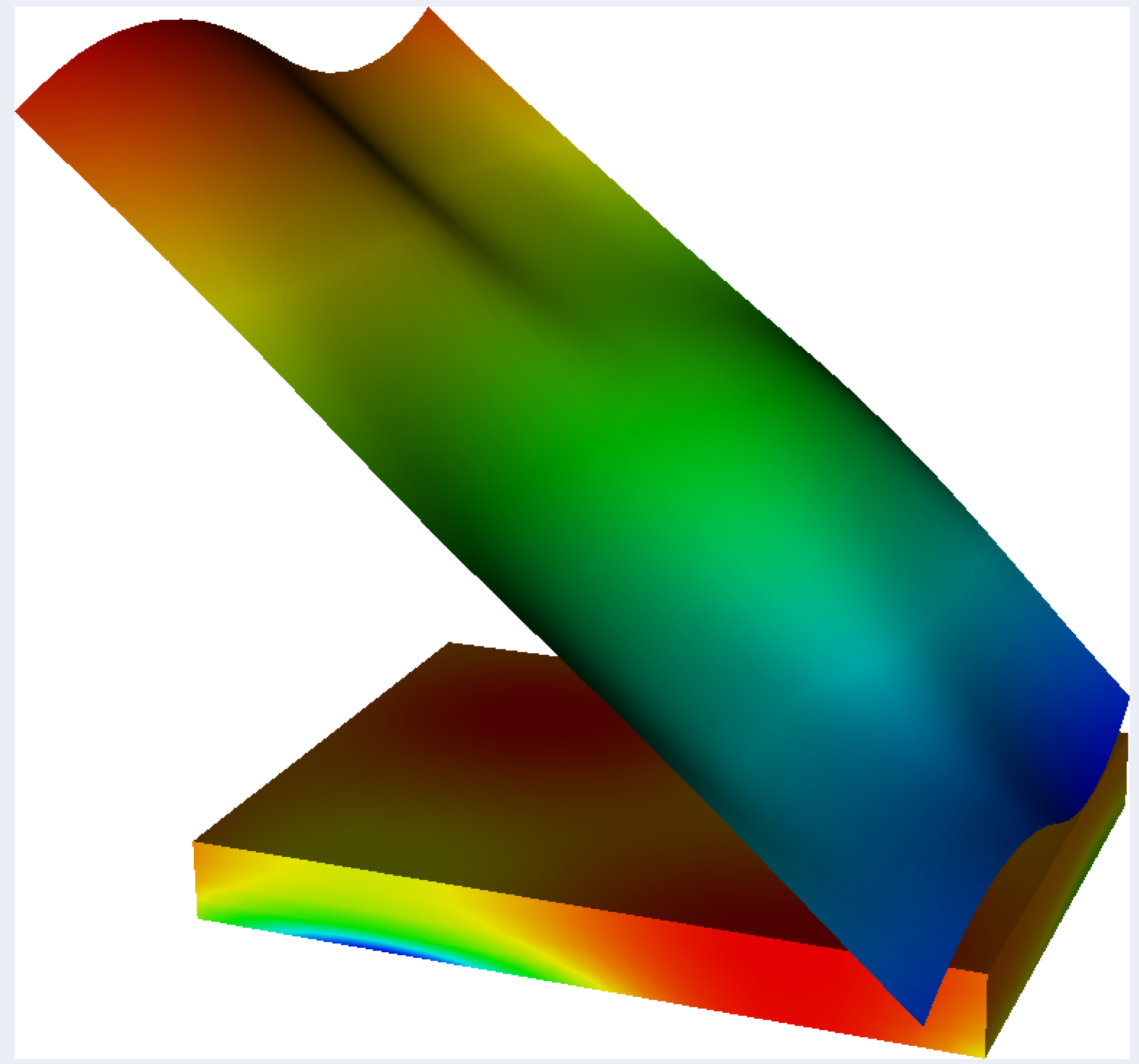

Figure: A steady-state solution for ISMIP-HOM test C [3] at 10km computed in 19 iterations. The elevated surface is exaggerated surface height.

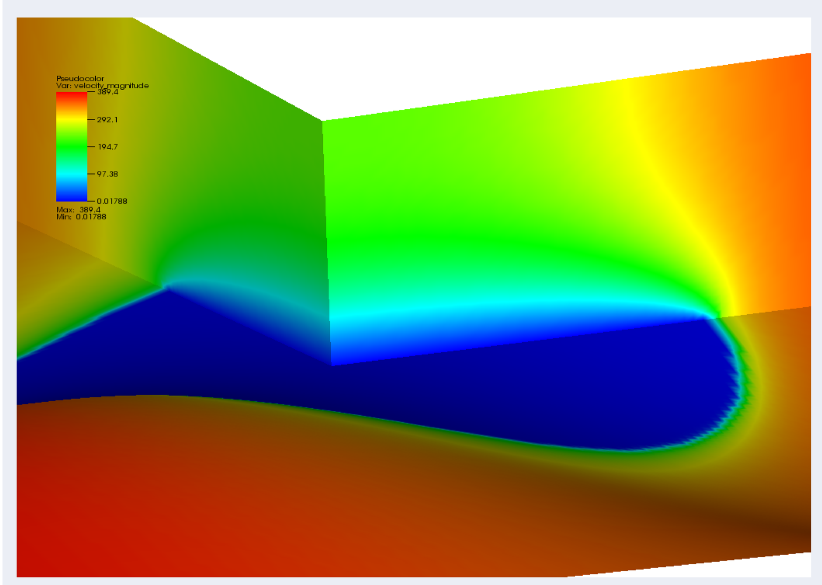

Figure: Shear margin for flow over a smooth bumpy bed with discontinuous sliding parameters and $m=1/10$ nearly plastic yield model.

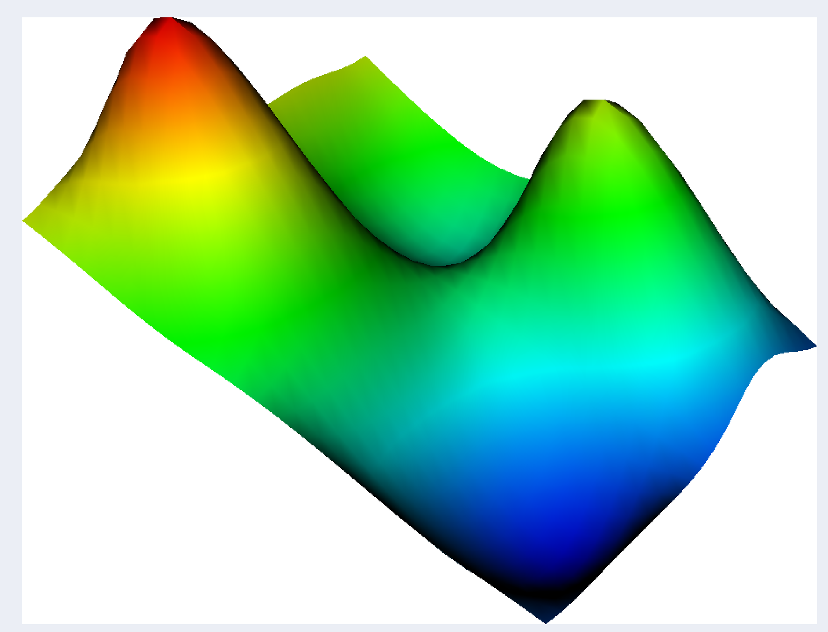

Figure: Bed profile eroded from a flat bed after 300 ka with test C slipperiness perturbation. Time steps are 30 ka at this point in the simulation.

Tightly coupled solvers, loosely coupled software

It is desirable to reuse exactly the same "physics" code to run single-physics models, coupled models using semi-implicit methods, fully implicit coupled models using split preconditioners, and fully implicit coupled models using monolithic preconditioners. A generic interface has been added to PETSc [1] with help from Dave May, for which the present code is a prototype client. It provides efficient assembly with arbitrary subphysics nesting and parallel decomposition, using a natural interface for "local submatrices". When the local submatrix interface is used for assembly, subphysics modules can be composed without recompilation into arbitrarily deep hierarchies, the hierarchy is flattened by the library so that performance is not affected.

The matrix format behind the interface can be chosen at runtime and includes a single monolithic matrix or nested pieces intended for use with field-split preconditioning with no memory or scalability penalty. Each nested piece can take advantage of efficient blocked and symmetric storage formats, offering performance gains of a factor of 2 for sparse matrix kernels and assembly. Matrix-free methods are fully supported and can be used for some or all physics components and inter-physics couplings.

Parallel Scalability on Blue Gene/P

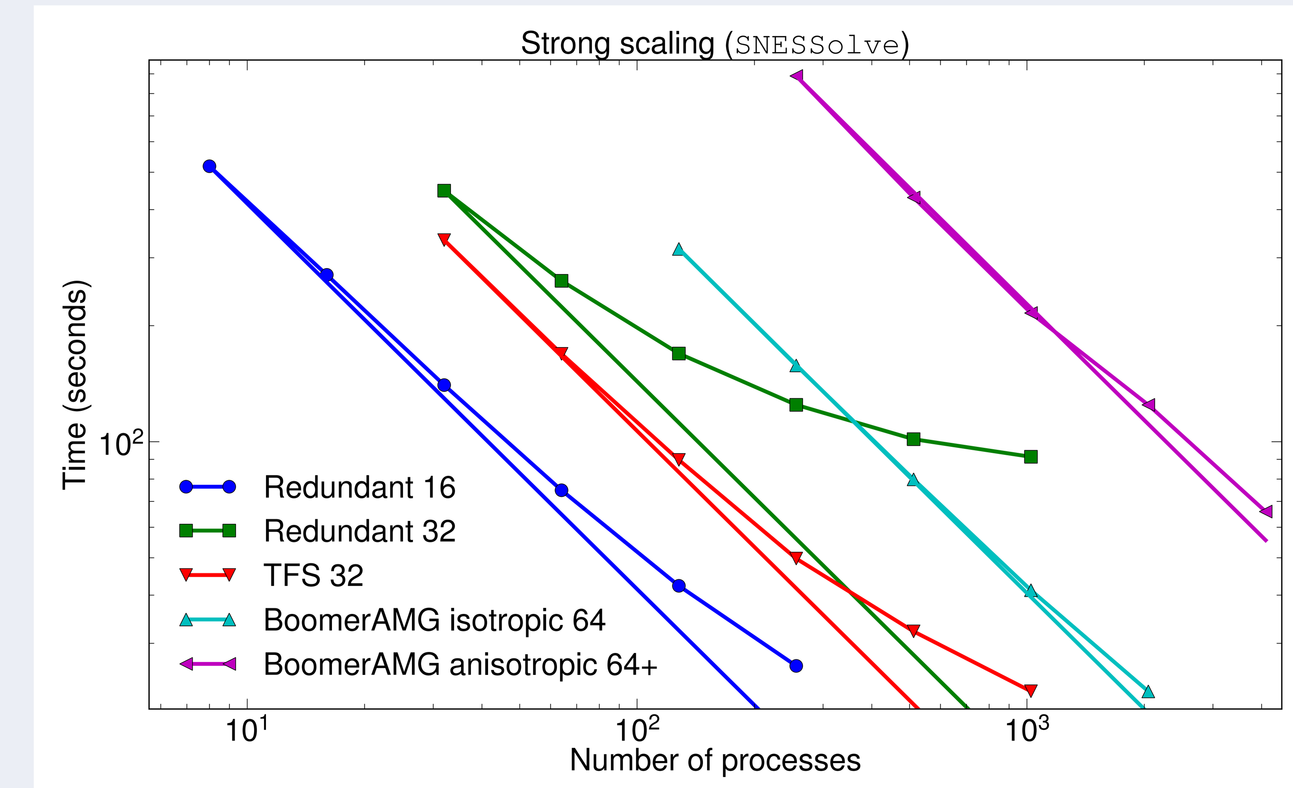

Figure: Strong scalability on Shaheen for different problem sizes with different coarse level solvers.

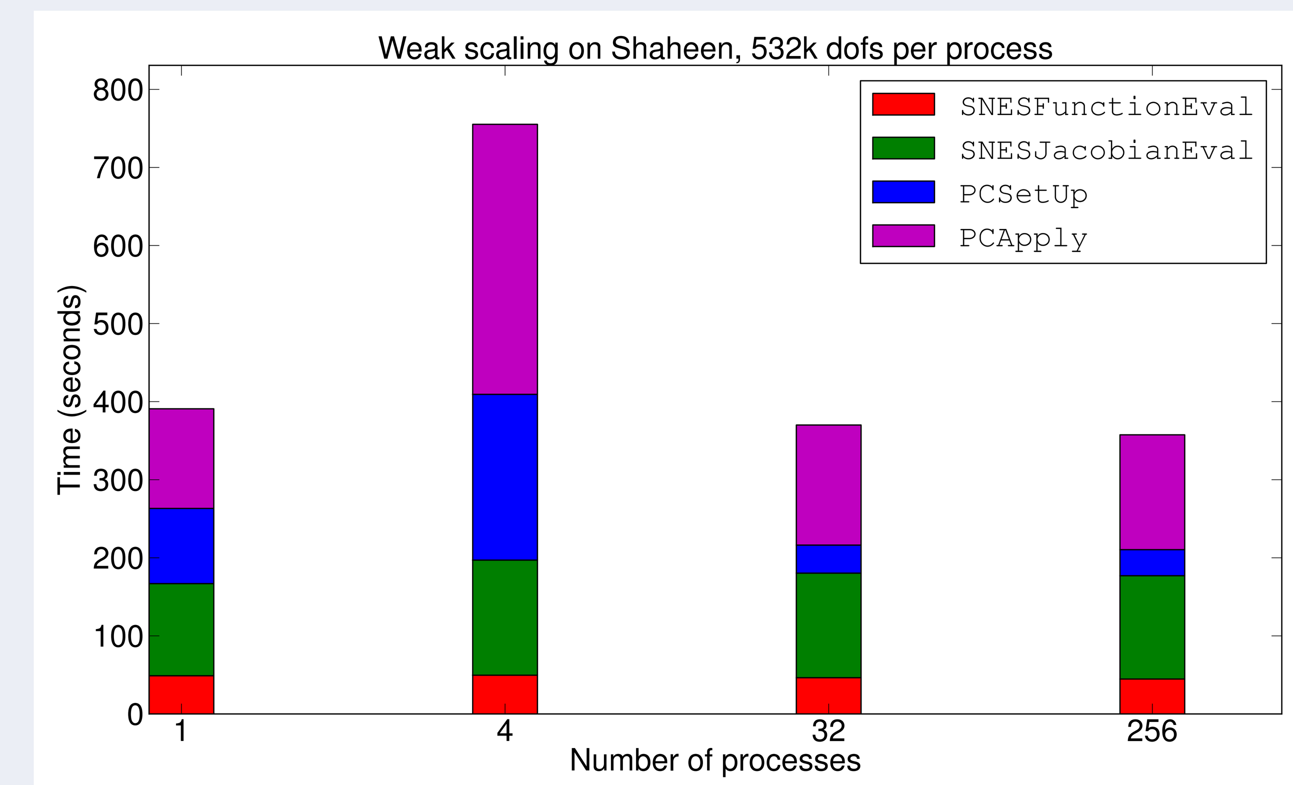

Figure: Weak scalability for an entire grid-sequenced solve of a problem with $m=1/5$ nonlinear sliding with discontinuous coefficients over a bumpy bed. Subdomains of size 64^3.

Discussion

- Steady state solves and long time steps are possible with fully implicit methods.
- Have a robust solver for the pure velocity system, thickness-velocity coupled linear solve is still hard for large or infinite time steps, need to treat with approximate Schur-complement methods.
- A similar model using the methods in [2] for 3D Stokes is in development, but geometric coupling in the ALE formulation is more difficult.
- Arc-length continuation and grid sequencing for coupled problems is underway.
- All software is available as part of the PETSc distribution or from myself.

References

[1] Satish Balay, Jed Brown, Kris Buschelman, William D. Gropp, Dinesh Kaushik, Matthew G. Knepley, Lois Curfman McInnes, Barry F. Smith, and Hong Zhang. PETSc Web page, 2010. mcs.anl.gov/petsc.

[2] Jed Brown. Efficient nonlinear solvers for nodal high-order finite elements in 3d. *Journal of Scientific Computing*, 45:48–63, 2010. 10.1007/s10915-010-9396-8.

[3] F. Pattyn, L. Perichon, A. Aschwanden, B. Breuer, B. De Smedt, O. Gagliardini, GH Gudmundsson, R. Hindmarsh, A. Hubbard, JV Johnson, et al. Benchmark experiments for higher-order and full Stokes ice sheet models (ISMIP-HOM). *The Cryosphere*, 2(1):95–108, 2008.

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